Bipartite graph embeddings, Riemann surfaces and Galois groups

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Bipartite graphs can model matching problems, hypergraphs and relations between sets. Studying their surface embeddings is an old tradition, illustrated by the utilities problem. I shall show how such embeddings can be described by pairs of permutations, and how this leads to a classification of the regular (most symmetric) embeddings of complete bipartite graphs. This is joint work with Du, Kwak, Nedela and Škoviera, building on earlier group-theoretic results of Hall, Huppert and Wielandt. Surface embeddings of bipartite graphs (called *dessins d'enfants* by Grothendieck) also give a link between compact Riemann surfaces and algebraic number fields, providing a faithful representation of the Galois group of the field of algebraic numbers, an important profinite group. I shall describe joint work with Streit and Wolfart on how this group acts on regular embeddings of complete bipartite graphs.