Ore-type condition for spanning trees with k vertices of degree 2

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In this talk, we only deal with simple undirected graphs. Let $\delta(G)$ and $\Delta(G)$ denote the minimum and maximum degree of a graph G, respectively, and let $\sigma_2(G)$ denote the minimum degree sum of nonadjacent vertices in G. Dirac [1] found sufficient conditions for a graph G to have a Hamilton cycle in terms of $\delta(G)$, and Ore [2] found similar conditions in terms of $\sigma_2(G)$. Ore's result follows that, for a graph G of order n, if $\sigma_2(G) \geq n-1$, then G has a Hamilton path. In this talk, we generalize the result by focusing the number of vertices of degree 2 in spanning trees.

Let G be a graph of order n such that $\sigma_2(G) \geq n-1$. Ore's result grantees that G has a spanning tree with $n-2$ vertices of degree 2. Our question is that, for an integer k $(0 \leq k \leq n-2)$, does G have a spanning tree with k vertices of degree 2? It is easy to see that G never has such a spanning tree when $k = n-3$. For other integers, we prove the following.

Theorem 1 *Let G be a graph of order n where $n \geq 10$ and let $k \in \{0, 1, \dots, n-4, n-2\}$ be an integers. If $\sigma_2(G) \geq n-1$, then, for each k, G has a spanning tree with k vertices of degree 2.*

In order to prove Theorem 1, we prove two lemmas. A *spider* is a tree obtained from a star by subdividing some edges (see Figure 1).

Lemma 1 *Let G be a graph of order n and let $k \in \{n-\Delta(G)-1 \dots, n-4, n-2\}$ be an integers. If $\sigma_2(G) \geq n-1$, then, for each k, G has a spider with k vertices of degree 2 as a spanning tree.*

Lemma 2 *Let G be a graph of order n where $n \geq 10$ and let $k \in \{0, 1 \dots, n-\Delta(G)-2\}$ be an integers. If $\sigma_2(G) \geq n-1$, then, for each k, G has a spanning tree with k vertices of degree 2.*

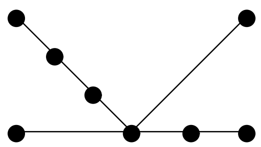

Figure 1: A spider obtained from $K_{1,4}$.

References

[1] G. Dirac, Some theorems on abstract graphs, Proc. London Math. Soc. 2 (1952), 69–81.

[2] O. Ore, A note on Hamilton circuits, Amer. Math. Monthly 67 (1960), 55.