Generating nonisomorphic maps and hypermaps without storing them and an application to group theory

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In 1979, while working as a senior researcher in the Computing Centre of the USSR Academy of Sciences in Moscow, I used A.B. Lehman's code for rooted maps of any orientable genus to generate these maps. By imposing an order on the code-words and keeping only those that are maximal over all the words that code the same map with each semi-edge chosen as the root, I generated these maps up to orientation-preserving isomorphism, and by comparing each of them with the code-words for the map obtained by reversing the orientation, I generated these maps up to a generalized isomorphism that could be orientation-preserving or orientation-reversing. The limitations on the speed of the computer I was using and the time allowed for a run restricted me to generating these maps with up to only six edges. In 2011, by optimizing the algorithms and using a more powerful computer and more CPU time I was able to generate these maps with up to eleven edges. An average-case time-complexity analysis of the generation algorithms is included. In 2012, by using a genus-preserving bijection between hypermaps and bicoloured bipartite maps that I discovered in 1975 and the condition on the word coding a rooted map for the map to be bipartite, I generated hypermaps, both rooted and unrooted, with up to twelve darts.

Using a bijection between rooted maps with e edges and subgroups of index $2e$ of Grothendiek's oriented cartographic group, and between maps up to orientation-preserving isomorphism and the conjugacy classes of these subgroups, Alain Giorgetti and I collaborated to generate these classes of subgroups for larger e than was possible previously with the same computing resources, and we can also generate all the subgroups, which was not possible previously. I implemented in C and in Magma the map generation algorithms and the conversion of a rooted map into a set of generators of a subgroup, and Giorgetti implemented in Magma the algorithm for constructing a subgroup from a set of generators and ran the completed program.