Total weight choosability of graphs

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A (proper) total weighting of a graph G is a mapping $\phi : V(G) \cup E(G) \to R$ such that any edge uv of G, $\sum_{e \in E(u)} \phi(e) + \phi(u) \neq \sum_{e \in E(v)} \phi(e) + \phi(v)$. A total weighting ϕ with $\phi(v) = 0$ for all vertices v is called an edge weighting. The well-known 1-2-3 conjecture says that every graph with no isolated edges has a 3-edge weighting, i.e., an edge weighting using weights 1,2,3. Berry, Dalal, McDiarmid, Reed, and Thomason proved that every graph with no isolated edges has a 30-edge weighting. This upper bound 30 was reduced subsequently to 16 (by Addario-Berry, Dalal, and Reed), 13 (by Wang and Yu) and 5 (by Kalkowski, Karoński, and Pfender). Przybyło and Woźniak conjectured that every graph has a 2-total weighting, and proved that every graph has a 11-total weighting. Kalkowski made a breakthrough and proved that every graph G has a total weighting ϕ with $\phi(v) \in \{1, 2\}$ for $v \in V(G)$ and $\phi(e) \in \{1, 2, 3\}$ for $e \in E(G)$. This talk discusses the list version of total weighting. A graph G is called (k, k')-choosable if for any list assignment L which assigns to each vertex k permissible weights, and to each edge k' permissible weights, there is a total weighting ϕ with $\phi(v) \in L(v)$ and $\phi(e) \in L(e)$, for $v \in V(G)$ and $e \in E(G)$. Wong and Zhu conjectured that every graph with no isolated edges is $(1, 3)$-choosable, and every graph is $(2, 2)$-choosable. This talk survey some progress on these conjectures. In particular, we shall prove that every graph is $(2, 3)$-choosable.