d-strong total colorings of graphs

Arnfried Kemnitz
(joint work with Massimiliano Marangio)

If $c : V \cup E \to \{1, 2, \ldots, k\}$ is a proper total coloring of a graph $G = (V, E)$ then the *palette* $S[v]$ of a vertex $v \in V$ is the set of colors of the incident edges and the color of v: $S[v] = \{c(e) : e = vw \in E\} \cup \{c(v)\}$. A total coloring c *distinguishes* vertices u and v if $S[u] \neq S[v]$. A *d-strong total coloring* of G is a proper total coloring that distinguishes all pairs of vertices u and v with distance $1 \leq d(u, v) \leq d$. The minimum number of colors of a d-strong total coloring is called *d-strong total chromatic number* $\chi''_d(G)$ of G. Such colorings generalize strong total colorings and adjacent strong total colorings as well. The d-strong total chromatic number is monotonous with respect to the distance and additive but not hereditary. Let n_i denote the maximum number of vertices of degree i that are of pairwise distance at most d and let $\mu''_d(G) = \max\{\min\{j : \binom{j}{i+1} \geq n_i\} : \delta(G) \leq i \leq \Delta(G)\}$. Obviously, $\mu''_d(G)$ is a lower bound for $\chi''_d(G)$. It was conjectured that $\mu''_d(G) + 1$ is an upper bound for the d-strong total chromatic number. We prove that this conjecture is not true in general. Moreover, we show that the difference between the d-strong total chromatic number $\chi''_d(C_n)$ of a cycle C_n and the lower bound $\mu''_d(C_n)$ may be arbitrarily large. In addition, we determine some general bounds for $\chi''_d(G)$, determine $\chi''_d(P_n)$ completely for paths P_n, give some exact values for the d-strong total chromatic number of cycles, and present results for circulant graphs.