# $C_3$-free vertices in almost regular 3-partite tournaments

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(joint work with Mika Olsen and Rita Zuazua)

The structure of cycles in multipartite tournaments has been extensively studied. A survey on this topic [5] appeared in 2007 with several interesting open problems. For instance, the study of cycles whose length does not exceed the number of partite sets leads to various extensions and gereralizations of classic results on tournaments. Bondy [1] proved that each strongly connected $c$-partite tournament contains an $m$-cycle for each $m \in \{3, \ldots, c\}$. In 1994, Guo and Volkmann [4] proved that every partite set of a strongly connected c-partite tournament $T$ has at least one vertex that lies on a cycle of length $m$ for each $m \in \{3, \ldots, c\}$. There are examples showing that not every vertex of a strongly connected $c$-partite tournament is contained in a cycle of length $m$ for each $m \in \{3, \ldots, c\}$ in general [5]. However, Zhou et al. [6] proved that every vertex of a regular $c$-partite tournaments with at least four parts is contained in a cycle of length $m$ for each $m \in \{3, \ldots, c\}$. Volkmann [5] provided an example that shows that this is not valid for regular 3-partite tournaments in general.

Let $T$ be a 3-partite regular tournament. We say that a vertex $v$ is $\overrightarrow{C_3}$-*free* if $v$ does not lie on any directed triangle of $T$. Let $F_3(T)$ be the set of the $\overrightarrow{C_3}$-free vertices in a 3-partite tournament. In 2010 Figueroa et al [2] proved that if $T$ is a regular 3-partite tournament, there is at most one partite set of $T$ containing the vertices in $F_3(T)$ and a bound of $|F_3(T)|$ was given. In 2012 Figueroa and Olsen [3] proved that $|F_3(T)| \leq \lfloor V(T)/12 \rfloor$ and generalized the Volkmann's family to show the tightness of this bound.

This talk is about the cardinality of $F_3(T)$ in almost regular 3-partite tournaments and results concerning this problem in 4-partite almost regular tournaments.

## References

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