Dense regular graphs: cycles and robust components

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We describe the large-scale structure of dense regular graphs [3]. This involves the notion of robust expansion, a recent concept which has already been used successfully to settle several longstanding problems, such as Kelly's conjecture [2]. Roughly speaking, a graph is robustly expanding if it still expands after the deletion of a small fraction of its vertices and edges.

Our main result states that every dense regular graph can be partitioned into 'robust components', each of which is a robust expander or a bipartite robust expander. We apply our result to obtain the following.

(i) We prove that whenever $\varepsilon > 0$, every sufficiently large 3-connected D-regular graph on n vertices with $D \geq (1/4 + \varepsilon)n$ is Hamiltonian. This asymptotically confirms the only remaining case of a conjecture raised independently in the 1970's by Bollobás [1] and Häggkvist.

(ii) We prove an asymptotically best possible result on the circumference of dense regular graphs of given connectivity. The 2-connected case was conjectured by Bondy and proved by Wei.

References

[1] B. Bollobás, Extremal graph theory, Dover, 2004, p. 167.

[2] D. Kühn, D. Osthus, Hamilton decompositions of regular expanders: a proof of Kelly's conjecture for large tournaments, Adv. Math. 237 (2013), 62–146.

[3] D. Kühn, A. Lo, D. Osthus, K. Staden, The robust component structure of dense regular graphs and applications, preprint.