Structure of factor-critical equimatchable graphs

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(joint work with Michal Kotrbčík)

A graph G is equimatchable if any matching of G is a subset of a maximum-size matching. From a general description of equimatchable graphs in terms of Gallai-Edmonds decomposition given in [2] follows that any 2-connected equimatchable graph is either bipartite or factor-critical. In both cases, the Gallai-Edmonds decomposition gives little additional information about the structure of such graphs. It is easy to see that for any vertex v of a factor-critical equimatchable graph G and a minimal matching M that isolates v the components of the graph $G \setminus (M \cup \{v\})$ are all either complete or regular complete bipartite. We proof that for any 2-connected factor-critical equimatchable graph G, the graph $G \setminus (M \cup \{v\})$ has at most one component. The structure of factor-critical equimatchable graphs with a cut-vertex and 2-cut is investigated in [1]. We extend these results and for all $k \geq 3$ we describe the structure of factor-critical equimatchable graphs with a k-vertex-cut. For $k \geq 3$, we prove that if a k-connected equimatchable factor-critical graph G with a k-vertex cut S has at least $2k+3$ vertices, or if $G \setminus S$ has a component with at least k vertices, then $G \setminus S$ has precisely 2 components. Moreover, if both these components have at least k vertices, then they are isomorphic with K_p, respectively K_q for some p and q. Our methods can be used to describe the structure of $G \setminus S$ also in the remaining cases, then, however, $G \setminus S$ can have many small components.

References

[1] O. Favaron, Equimatchable factor-critical graphs, J. Graph Theory 10:4 (1986), 439–448.

[2] M. Lesk, M.D. Plummer, W.R. Pulleyblank, Equimatchable graphs. in: B. Bollobás (ed.), Graph theory and combinatorics, Proc. Cambridge Combin. Conf. in Honour of Paul Erdős, Academic Press, 1984, pp. 239–254.