On computing an optimal semi-matching

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The problem of finding an optimal semi-matching is a generalization of the problem of finding classical matching in bipartite graphs. A *semi-matching* in a bipartite graph $G = (U, V, E)$ with n vertices and m edges is a set of edges $M \subseteq E$, such that each vertex in U is incident to at most one edge in M. An *optimal semi-matching* is a semi-matching with $deg_M(u) = 1$ for all $u \in U$ and the minimal value of $\sum_{v \in V} \frac{deg_M(v).(deg_M(v)+1)}{2}$ (see e.g. [2]). We propose a schema that allows a reduction of the studied problem to a variant of the maximum bounded-degree semi-matching problem. The proposed schema yields to two algorithms for computing an optimal semi-matching. The first one runs in time $O(\sqrt{n} \cdot m \cdot \log n)$ that is the same as the time complexity of the currently best known algorithm (see [1]). However, our algorithm uses a different approach that enables some improvements in practice (e.g. parallelization, faster algorithms for special graph classes). The second one is randomized and it computes an optimal semi-matching with high probability in $O(n^\omega \cdot \log^{1+o(1)} n)$, where ω is the exponent of the best known matrix multiplication algorithm. Since $\omega \leq 2.38$, this algorithms breaks through $O(n^{2.5})$ barrier for dense graphs. The core of the presented approach was developed in [3].

References

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