

Statues and double branch weights of some outerplanar graphs

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(joint work with Halina Bielak)

Let $G = (V(G), E(G))$ be a graph. Let $w : V(G) \cup E(G) \rightarrow R^+$ be a weight function. The pair (G, w) is called a *weighted graph*. For a path P in G , the *weight length* of P , denoted by $l_w(P)$, is defined by $l_w(P) = \sum_{e \in E(P)} w(e)$. For vertices x, y in G , the *weight distance* between x and y , denoted by $d_w(x, y)$, is defined as $d_w(x, y) = \min\{l_w(P)\}$, where the minimum is taken over all paths P joining x and y .

For any vertex x of G , the *status* of x , denoted by $s(x)$, is defined by $s(x) = \sum_{y \in V(G)} w(y)d_w(y, x)$. The *median* of G , denoted by $M_1(G)$, is the set of vertices in G with the smallest status. The *second median* of G , denoted by $M_2(G)$, is the set of vertices in G with the second smallest status. The *weight of the graph* G , denote by $w(G)$, is defined by $w(G) = \sum_{x \in V(G)} w(x)$. Let $S \subseteq V(G)$. By $G - S$ we denote the graph obtained from G by deleting all vertices of S with the edges incident to S .

In this paper we show some metric properties of a family of weighted outerplanar graphs. For an outerplanar graph G we define some additional notions and notations. Namely, for any two vertices x and y of G , where $\{x, y\} \in E(G)$, we define the *edge double branch weight* of an edge $\{x, y\}$, denoted by $dbw(\{x, y\})$, as the maximum weight of any component of $G - \{x, y\}$.

The *double branch weight of outerplanar graph* G , denoted by $DBW(G)$, is defined by $DBW(G) = \min\{dbw(\{x, y\}) : x, y \in V(G), \{x, y\} \in E(G)\}$. The *vertex double branch weight* of a vertex x we define as follows $dbw^*(x) = \min\{dbw(\{x, y\}) : \{x, y\} \in E(G)\}$.

The *first double centroid of outerplanar graph* G , denoted by $DC_1(G)$, is the set of vertices in G with the smallest vertex double branch weight. The *second double centroid of outerplanar graph* G , denoted by $DC_2(G)$, is the set of vertices in G with the second smallest vertex double branch weight.

We present the lower and upper bounds for double branch weights of some outerplanar graphs. Moreover, we show some relations between the first median, the second median, the first double centroid and the second double centroid for some outerplanar graphs. In this way we extend some results of Lin and Shang [1] and Zelinka [2].

REFERENCES

- [1] Ch. Lin, J.-L. Shang, Statues and branch-weights of weighted trees, Czech. Math. J. 59 (2009), 1019–1025.
- [2] B. Zelinka, Medians and peripherians of trees, Arch. Math. (Brno) 4 (1968), 87–95.