Zero-one laws for minor-closed classes of graphs

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Let $\mathcal{G}$ be a class of labelled graphs endowed with a probability distribution on the set $\mathcal{G}_n$ of graphs in $\mathcal{G}$ with n vertices. We say that a zero-one law holds in $\mathcal{G}$ if every first order graph property holds or does not hold in $\mathcal{G}_n$ with probability 1 as n goes to infinity. Many zero-one laws have been established for the classical binomial model $G(n, p)$ of random graphs, as well as for other classes such as random regular graphs. In this talk we present a zero-one law for connected graphs in a class of graphs $\mathcal{G}$ closed under taking minors, with the property that all forbidden minors of $\mathcal{G}$ are 2-connected. Interesting classes of this kind include trees and planar graphs. A zero-one law does not hold for non-necessarily connected graphs in $\mathcal{G}$ as, for instance, the probability of having an isolated vertex tends to a constant strictly between 0 and 1. For arbitrary graphs in $\mathcal{G}$ we prove a convergence law, that is, every first order property has a limiting probability. These results hold more generally for properties expressible in monadic second order logic. On the other hand, given a fixed surface S, we prove a convergence law in first order logic for the class of graphs embeddable in S (this class is closed under minors but the forbidden minors are not necessarily 2-connected). Moreover, we prove that the limiting probabilities of first order properties do not depend on S.