Trinal decompositions of Steiner triple systems

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(joint work with Curtis Lindner and Alexander Rosa)

Let STS(n) be a Steiner triple system of order n. A *triangle* T is a set of three pairwise intersecting triples of an STS(n) whose intersection is empty.

Z. Füredi posed a question whether the set of triples of any STS(n) can be decomposed into triangles. This question remains largely unanswered, although examples of such decomposition are known for every admissible order $n \equiv 1$ or 9 (*mod* 18).

A triangle $T = \{\{a, b, c\}, \{c, d, e\}, \{e, f, a\}\}$ in an STS(n) is sometimes called a *hexagon triple* because the outer edges ab, bc, cd, de, ef, fa form a hexagon. Depending on a graph-theoretic or geometric representation, respectively, that triangle determines naturally two more triples: the *inner* triple $\{a, c, e\}$ and the *midpoint* triple $\{b, d, f\}$. In either case, the number of inner triples (called *type* 1) or that of midpoint triples (*type* 2) equals one third of the total number of triples in an STS(n).

A problem which will be discussed concerns the existence of three distinct decompositions of an STS(n) into triangles such that the union of three collections of type 1 triples (type 2, respectively) from these three decompositions form a set of triples of a Steiner triple system of the same order n. Such decompositions are called *trinal* decompositions of type 1 and type 2, respectively.

Solutions to the existence question for trinal decompositions of type 1 and type 2 will be presented.