# On the family of $r$-regular graphs with Grundy number $r+1$

**Nicolas Gastineau**

(joint work with Hamamache Kheddouci and Olivier Togni)

The *Grundy number* of a graph $G$, denoted by $\Gamma(G)$, is the largest $k$ such that there exists a partition of $V(G)$, into $k$ independent sets $V_1, \ldots, V_k$ and every vertex in $V_i$ is adjacent to at least one vertex in $V_j$, for every $j < i$. Let the neighborhood $N(v)$ of a vertex $v$ be $\{u \in V(G) | uv \in E(G)\}$. A set $X$ of vertices is an *independent module* if $X$ is an independent set and all vertices in $X$ have the same neighborhood.

The Grundy number is a well studied problem [1, 2, 5]. The $b$-chromatic number of regular graphs has been investigated in a series of papers [3, 4]. Our aim is to establish similar results for the Grundy number. Our subject is the family of $r$-regular graphs such that $\Gamma(G) = r + 1$. Using the notion of independent module, a characterization of this family is given for $r = 3$. Moreover, we prove that, for $r \leq 4$, the family of $r$-regular graphs without induced $C_4$ is included in this family.

**Definition 1** *Let $G$ be an $r$-regular graph.*

1. *A vertex $v$ is a $(0, \ell)$-twin-vertex if there exists an independent module of cardinality $r + 2 - \ell$ which contain $v$.*
2. *A vertex $v$ is a $(1, \ell)$-twin-vertex if $N(v)$ can be partitioned into at least $\ell - 1$ independent modules.*
3. *A vertex $v$ is a $(2, \ell)$-twin-vertex if $N(v)$ is independent and every vertex in $N(v)$ is a $(1, \ell)$-twin-vertex.*

**Theorem 2** *Let $G$ be a cubic graph. $\Gamma(G) \leq 3$ if and only if every vertex is a $(i, 3)$-twin-vertex, for some $i$, $0 \leq i \leq 2$.*

**Theorem 3** *Let $G$ be a $r$-regular graph, with $r \leq 4$. If $G$ does not contain an induced $C_4$, then $\Gamma(G) = r + 1$.*

**Conjecture 4** *For any $r \geq 1$, every $r$-regular graph without induced $C_4$ has Grundy number $r + 1$.*

**Theorem 5** *Let $r \geq 4$ and $3 \leq k \leq r + 1$ be integers. There exists a infinite family $\mathcal{F}$ of $r$-regular graphs such that $\forall G \in \mathcal{F}$, $\Gamma(G) = k$.*

## References

[1] G.J. Chang, H. Hsu, First-fit chromatic numbers of $d$-degenerate graphs, Discrete Math. 312 (2012), 2088–2090.

[2] F. Havet, L. Sampaio, On the Grundy and $b$-chromatic numbers of a graph, Algorithmica 65 (2013), 885–899.

[3] M. Jakovac, S. Klavžar, The $b$-chromatic number of cubic graphs, Graphs Combin. 26 (2010), 107–118.

[4] S. Shaebani, On the $b$-chromatic number of regular graphs without 4-cycle, Discrete Appl. Math. 160 (2012), 1610–1614.

[5] M. Zaker, Results on the Grundy chromatic number of graphs, Discrete Math. 306 (2006), 3166–3173.