New lower bounds on the maximum number of arc-disjoint cycles in a digraph

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N. Alon, C. McDiarmid and M. Molloy, proved in [2] that for $d \geq 1$, any d-regular digraph contains at least $2.5d-2$ arc-disjoint cycles. In this paper, first we prove that for $d \leq 4$, the result above is in fact valid for any digraph of minimum out-degree d. In other words, we prove that a digraph of minimum out-degree 2 contains at least 3 arc-disjoint cycles, that a digraph of minimum out-degree 3 contains at least 6 arc-disjoint cycles, and that a digraph of minimum out-degree 4 contains at least 8 arc-disjoint cycles. In a second part, by using the result of the first section, and a result on the maximum number of vertex-disjoint cycles in a digraph with minimum out-degree 5 (see [3]), we prove that for $d \geq 4$, any digraph of minimum out-degree d, contains at least $3d-4$ arc-disjoint cycles. So our lower bound is better than that of Alon et all, and concerns a larger class of digraphs. Alon proved also that a digraph of minimum out-degree d contains at least $\frac{d^2}{128}$ arc-disjoint cycles (see [1]). A simple verification shows that for $d \leq 382$, our lower bound improves that of Alon.

References

[1] N. Alon, Disjoint directed cycles, J. Combin. Theory Ser. B, 68:2 (1996), 167–178.

[2] N. Alon, C. McDiarmid, M. Molloy, Edge disjoint cycles in regular directed graphs, J. Graph Theory 22:3 (1996), 231–237.

[3] N. Lichiardopol, A. Pór, J.-S. Sereni, A step toward the Bermond-Thomassen conjecture about disjoint cycles in digraphs, SIAM J. Discrete Math. 23:2 (2009), 979–992.