Asymptotically optimal neighbour sum distinguishing colourings of graphs

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Consider a simple graph $G = (V, E)$ and its *proper* edge colouring c with the elements of the set $\{1, 2, \ldots, k\}$. The colouring c is said to be *neighbour sum distinguishing* if for every pair of vertices u, v adjacent in G, the sum of colours of the edges incident with u is distinct from the corresponding sum for v. The smallest integer k for which such colouring exists is known as the *neighbour sum distinguishing index* of a graph and denoted by $\chi'_{\sum}(G)$. The definition of this parameter, which makes sense for graphs containing no isolated edges, immediately implies that $\chi'_{\sum}(G) \geq \Delta$, where Δ is the maximum degree of G. On the other hand, it was conjectured by Flandrin et al. that $\chi'_{\sum}(G) \leq \Delta + 2$ for all those graphs, except for C_5. We prove this bound to be asymptotically correct by showing that $\chi'_{\sum}(G) \leq \Delta(1 + o(1))$. The main idea of our argument relays on a random assignment of the colours, where the choice for every edge is biased by so called *attractors*, randomly assigned to the vertices.