

Avoiding 7-circuits in 2-factors of cubic graphs

Robert Lukotřka

Jaeger and Swart conjectured that there is no cyclically 7-edge-connected snark [1]. This implies that every cyclically 7-edge-connected cubic graph has an even 2-factor. The conjecture is widely open. To start the investigation we focus on 7-circuits.

Let G be a cyclically 5-edge-connected cubic graph with girth 7. We show that G has a 2-factor F such that at least a linear amount of vertices of G is outside 7-circuits of F . For a cyclically 7-edge-connected graph we obtain a better constant. This result gives first non-trivial lower bound on the oddness ratio $V(G)/\omega(G)$ of a cyclically 6-edge connected cubic graph [2].

Finally, for a cyclically 7-edge connected snark G we prove the following weakening of Jaeger-Swart conjecture. We prove that given one vertex v of G , we can choose F so that v is not in a 7-circuit of F . Note that the above observation is not true for cyclically 5-edge-connected snarks and 5-circuits because the Petersen graph has only 5-circuits in all of its 2-factors. Therefore no analogue to Petersen graph in this sense exists.

REFERENCES

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- [2] R. Lukotřka, E. Máčajová, J. Mazák, M. Škoviera, Small snarks with large oddness, arXiv:1212.3641.