Signed homomorphisms of planar signed graphs to signed projective cubes

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(joint work with Reza Naserasr and Éric Sopena)

A signed graph (G, Σ) is a graph G together with an assignment of signs $+$ and $-$ to all the edges of G where Σ is the set of negative edges. Furthermore (G, Σ_1) and (G, Σ_2) are considered to be equivalent if the symmetric difference of Σ_1 and Σ_2 is an edge cut of G [1].

Given two signed graphs (G, Σ_1) and (H, Σ_2), we say that there is a signed homomorphism of (G, Σ_1) to (H, Σ_2) if there is a signed graph (G, Σ_1') equivalent to (G, Σ_1) and a signed graph (H, Σ_2') equivalent to (H, Σ_2) together with a mapping $\phi : V(G) \to V(H)$ such that every edge of (G, Σ_1') is mapped to an edge of (H, Σ_2') of the same sign.

In the talk we will investigate signed homomorphisms of planar signed bipartite graphs to signed projective cubes. The projective cube of dimension d, denoted $\mathcal{P}C_d$, is the Cayley graph $(\mathbb{Z}_2^d, \{e_1, e_2, \ldots, e_d\} \cup \{J\})$ where e_i is the vector of $\mathbb{Z}_2^d$ with the i-th coordinate being 1 and other coordinates being 0 and $J = (1, 1, 1, \ldots, 1)$. Let $\mathcal{J}$ be the set of edges corresponding to J. Then the signed projective cube of dimension d is the signed graph $(\mathcal{PC}_d, \mathcal{J})$.

We conjecture [2] that if G is a planar bipartite graph and Σ is a signature such that the shortest unbalanced cycle (i.e. a cycle with odd number of negative edges) of (G, Σ) is of length at least $2g$, then (G, Σ) admits a homomorphism to the signed projective cube of dimension $2g-1$. We will show that this conjecture is equivalent to the corresponding case of a conjecture of Seymour [3] stating that every planar $2g$-regular multigraph with no odd edge-cut of less than $2g$ edges is $2g$-edge-colorable.

References

[1] R. Naserasr, E. Rollová, É. Sopena, Homomorphisms of signed graphs, preprint.
[2] R. Naserasr, E. Rollová, É. Sopena, Homomorphisms of planar signed graphs to signed projective cubes, preprint.
[3] P. Seymour, Matroids, hypergraphs and the Max.-Flow Min.-Cut Theorem, D. Phil. thesis, University of Oxford, 1975, p. 34.