Improved bounds on the chromatic numbers of the square of Kneser graphs

Seog-Jin Kim

(joint work with Boram Park)

The Kneser graph $K(n,k)$ is the graph whose vertices are the k-elements subsets of an n-element set, with two vertices adjacent if the sets are disjoint. The square G^2 of a graph G is the graph defined on $V(G)$ such that two vertices u and v are adjacent in G^2 if the distance between u and v in G is at most 2. Determining the chromatic number of the square of the Kneser graph $K(2k+1,k)$ is an interesting problem, but not much progress has been made. Kim and Nakprasit [2] showed that $\chi(K^2(2k+1,k)) \leq 4k+2$, and Chen, Lih, and Wu [1] showed that $\chi(K^2(2k+1,k)) \leq 3k+2$ for $k \geq 3$. In this paper, we give improved upper bounds on $\chi(K^2(2k+1,k))$. We show that $\chi(K^2(2k+1,k)) \leq 2k+2$, if $2k+1 = 2^n - 1$ for some positive integer n. Also we show that $\chi(K^2(2k+1,k)) \leq \frac{8}{3}k + \frac{20}{3}$ for every integer $k \geq 2$. In addition to giving improved upper bounds, our proof is concise and can be easily understood by readers while the proof in [1] is very complicated. Moreover, we show that $\chi(K^2(2k+r,k)) = \Theta(k^r)$ for each integer $2 \leq r \leq k-2$.

References

[1] J.-Y. Chen, K.-W. Lih, J. Wu, Coloring the square of the Kneser graph $KG(2k+1,k)$ and the Schrijver graph $SG(2k+2,2)$, Discrete Appl. Math. 157 (2009), 170–176.

[2] S.-J. Kim, K. Nakprasit, On the chromatic number of the square of the Kneser graph $K(2k+1,k)$, Graphs Combin. 20 (2004), 79–90.