On some Zarankiewicz numbers and bipartite Ramsey numbers for quadrilateral

Tomasz Dzido
(joint work with Janusz Dybizbański and Stanisław Radziszowski)

The Zarankiewicz number \( z(m,n; s,t) \) is the maximum number of edges in a subgraph of \( K_{m,n} \) that does not contain \( K_{s,t} \) as a subgraph. The bipartite Ramsey number \( b(n_1, \ldots, n_k) \) is the least positive integer \( b \) such that any coloring of the edges of \( K_{b,b} \) with \( k \) colors will result in a monochromatic copy of \( K_{n_i,n_i} \) in the \( i \)-th color, for some \( i, 1 \leq i \leq k \). If \( n_i = m \) for all \( i \), then we denote this number by \( b_k(m) \).

In this talk we obtain the exact values of some Zarankiewicz numbers for quadrilateral \( (s=t=2) \), and we derive new bounds for diagonal multicolor bipartite Ramsey numbers avoiding quadrilateral. In particular, we prove that \( b_4(2) = 19 \), and establish new general lower and upper bounds on \( b_k(2) \).

References