Resolvable edge decompositions of graphs

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(joint work with Selda Küçükçifçi and Salvatore Milici)

We study edge decompositions of the complete graph K_v of order v, and of the graph $K_v - I$ which is obtained from K_v by removing a 1-factor.

Given a collection $\mathcal{H}$ of graphs, an *$\mathcal{H}$-decomposition* of a graph G is a partition of the edge set of G into subgraphs isomorphic to members of $\mathcal{H}$. The subgraphs (copies of $H \in \mathcal{H}$) in the decomposition are called *blocks*. A decomposition is *resolvable* if the set of blocks can be partitioned into *classes* $\mathcal{P}_i$ in such a way that each vertex of G appears in exactly one block of each $\mathcal{P}_i$. A class is called *uniform* if all of its blocks are isomorphic to the same graph from $\mathcal{H}$.

In the current work we restrict our attention to the cases $G = K_v$ and $G = K_v - I$, and focus on $\mathcal{H} = \{K_{1,3}, K_3\}$, i.e. where $\mathcal{H}$ consists of the 3-edge star and the triangle. We study the existence of uniformly resolvable decompositions of those G into given numbers of classes which contain only copies of $K_{1,3}$ or K_3 (but not both).

Decompositions into other pairs of small graphs have also been studied by Rees, by Dinitz, Ling and Danziger, and by subsets of the authors of the present paper.